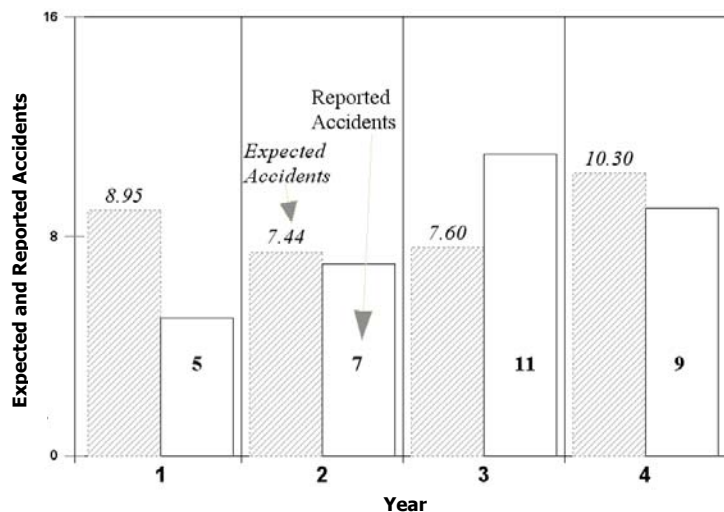


APPENDIX A—AVERAGE CRASH FREQUENCY ESTIMATION METHODS WITH AND WITHOUT HISTORIC CRASH DATA

This appendix provides a summary of additional methods for estimating crash frequency with and without crash data. These methods are a summary of findings from research conducted for NCHRP 17-27 and presented here for reference. The variables and terminology presented in this appendix are not always consistent with the material in Chapter 3.

The additional methods are presented through examples based on the hypothetical situation summarized in Exhibit A-1. This exhibit summarizes an intersection’s expected and reported accidents over a four-year period. The expected average crash frequency is shown in the shaded columns. The reported accident count for each year is shown in the un-shaded columns.

Exhibit A-1: Intersection Expected and Reported Accidents for Four Years



A.1 Statistical Notation and Poisson Process

The following notation is defined:

Reported accident count:

X = 'Accident count';

$X = x$ means that the 'Accident count' is some integer x ;

X_i = the subscript 'i' denotes a specific period, for example, in Exhibit A-1 $X_1=5$ for Year 1 and $X_2=7$ for Year 2;

Expected average crash frequency:

$E\{ \}$ = 'Expected value', for example, in Exhibit A-1 $E\{X_1\}$ is the expected average crash frequency in Year 1;

$E\{X_i\} \equiv \mu_i$, that is, the Greek letter μ has the same meaning as $E\{ \}$;

Variance :

28 $V\{X_i\} \equiv E\{(X_i - \mu_i)^2\}$ = the variance of X_i ;

29 $V\{X_i\} \equiv \sigma_i^2$

30 'Estimate of':

31 $\hat{\mu}_i$ = the estimate of μ_i ;

32 $\hat{\sigma}_i$ = the estimate of σ_i = the standard error of $\hat{\mu}_i$.

33 In statistics, the common assumption is that several observations are drawn from
 34 a distribution in which the expected value remains constant. Using the several
 35 observed values, the standard error of the estimate is computed.

36 In road safety, the expected average crash frequency from one period cannot be
 37 assumed to be and is not the same as that of another time period. Therefore, for a
 38 specific time period, only one observation is available to estimate μ . For the example
 39 in Exhibit A-1, the change from Year 1 to Year 2 is based on only one accident count
 40 to estimate μ_1 and one other accident count to estimate μ_2 .

41 Using one accident count per estimate seems to make the determination of a
 42 standard error impossible. However, this issue is resolved by the reasonable
 43 assumption that the manner of accident generation follows the Poisson process. The
 44 Poisson process is the most important example of a type of random process known as
 45 a 'renewal' process. For such processes the renewal property must only be satisfied at
 46 the arrival times; thus, the interarrival times are independent and identically
 47 distributed, as is the case for the occurrence of accidents.

48 The Poisson probability mass or distribution function is shown in Equation A-1.

49
$$P(X_i = x) = \frac{\mu_i^{(x)} \times e^{(-\mu_i)}}{x!} \tag{A-1}$$

50 Where,

51 μ_i = the expected number of accidents for a facility for period i ;

52 $P(X_i = x)$ = the probability that the reported number of accidents X_i for
 53 this facility and period 'i' is x ;

54 It is the property of the Poisson distribution that its variance is the same as its
 55 expected value, as shown in Equation A-2.

56
$$V\{X\} \equiv \sigma^2 = \mu \equiv E\{X\} \tag{A-2}$$

57 Where,

58 $V(X)$ = variance of $X = \sigma^2$

59 $\mu \equiv E\{X\}$ = expected average crash frequency .

60 **A.2 Reliability and Standard Error**

61 As all estimates are subject to uncertainty, the reliability of an estimate is
 62 required in order to know the relationship between the expected and reported values.
 63 This is why, as a rule, estimates are often accompanied by a description of their
 64 standard error, variance or some manner of statistical reliability.

65 The “standard error” is a common measure of reliability. Exhibit A-2 describes
 66 the use of the standard error in terms of confidence levels, i.e. ranges of closeness to
 67 the true value, expressed in numeric and verbal equivalents.

68 **Exhibit A-2: Values for Determining Confidence Intervals using Standard Error**

Desired Level of Confidence	Confidence Interval (probability that the true value is within the confidence interval)	Multiples of Standard Error (MSE) to using in Equation 3-8
Low	65-70%	1
Medium	95%	2
High	99.9%	3

69
 70 The estimates of the mean and the standard error if X is Poisson distributed are
 71 shown in Equation A-3.

$$\hat{\mu}_i = x \quad \text{and} \quad \hat{\sigma}_i = \sqrt{x} \tag{A-3}$$

73 Where,

74 $\hat{\mu}_i$ = the estimate of μ_i ;

75 x = accident count;

76 $\hat{\sigma}_i$ = the estimate of σ_i or the estimate of the standard error.

77 For example, the change between two time periods for the intersection in Exhibit
 78 A-1 can be estimated as follows:

$$\hat{\mu}_{year\ 1} = 5 \text{ accidents} \quad \text{and} \quad \hat{\sigma}_{year\ 1} = \pm 2.2 \text{ accidents}$$

80 The change between Year 1 to Year 2 is estimated by the difference between μ_{year2}
 81 and μ_{year1} . Using the first part of Equation A-3:

$$\hat{\mu}_{year\ 2} - \hat{\mu}_{year\ 1} = X_2 - X_1 = 7 - 5 = 2 \text{ accidents}$$

83 Since X_1 and X_2 are statistically independent, the variance of the change is as
 84 shown in Equation A-4.

$$V\{X_2 - X_1\} = \sigma_1^2 + \sigma_2^2 \tag{A-4}$$

86 Where,

87 X_i = accident count for specific period;

88 $\hat{\sigma}_i$ = the estimate of σ_i or the estimate of the standard error.

89 Using Equation A-3 and Equation A-4 in the example shown in Exhibit A-1, the
 90 standard error of the difference between Year 1 and Year 2 is:

$$\hat{\sigma}_{year\ 2} - \hat{\sigma}_{year\ 1} = \sqrt{5 + 7} = \pm 3.5 \text{ accidents}$$

92 In summary, the change between Year 1 and Year 2 is 2 accidents \pm 3.5 accidents.
 93 As indicated in Exhibit A-2, the standard error means we are:

- 94 ■ 65-70% confident that the change is in the range between -1.5 and +5.5
- 95 accidents ($2 - 3.5 = -1.5$, and $2 + 3.5 = +5.5$);
- 96 ■ 95% confident that the change is between is in the range between -5 and +9
- 97 accidents ($2 - 2 \times 3.5 = -5$, and $2 + 2 \times 3.5 = +9$);
- 98 ■ 99.9% confident that the change is in the range between -8.5 to 12.5 accidents.

99 If any one of these ranges was completely on one side of the value zero with zero
100 meaning no change, then an increase or decrease could be estimated with some level
101 of confidence. However, because the ranges are wide and encompass zero, the
102 expected increase of 2 accidents provides very little information about how changes
103 from year 1 to year 2. This is an informal way of telling whether an observed
104 difference between reported accidents counts reflects a real change in expected
105 average crash frequency.

106 The formal approach requires a statistical hypothesis which postulates that the
107 two expected values were not different.⁽⁹⁾ The observed data are investigated and if it
108 is concluded that the hypothesis of 'no difference' can be rejected at a customary level
109 of significance¹ ' α ' ($\alpha=0.05, 0.01, \dots$) then it may be reasonable to conclude that the
110 two expected values were different.

111 It is important to understand the results of statistical tests of significance. A
112 common error to be avoided occurs when the hypothesis of 'no difference' is not
113 rejected and an assumption is made that the two expected values are likely to be the
114 same, or at least similar. This conclusion is seldom appropriate. When the hypothesis
115 of no difference is 'not rejected' it may means that the accident counts are too small to
116 say anything meaningful about the change in expected values. The potential harm to
117 road safety management of misinterpreting statistical tests of significance is
118 discussed at length in other publications.⁽¹⁰⁾

119 **A.3 Estimating Average Crash Frequency**

120 **Based on Historic Data of One Roadway or**

121 **One Facility**

122 It is common practice to estimate the expected crash frequency of a roadway or
123 facility using a few, typically three, recent years of accident counts. This practice is
124 based on two assumptions:

- 125 ■ Reliability of the estimation improves with more accident counts;
- 126 ■ Accident counts from the most recent years represent present conditions
127 better than older accident counts.

128 These assumptions do not account for the change in conditions which occur on
129 this roadway or facility from period-to-period or year-to-year. There are always
130 period-to-period differences in traffic, weather, accident reporting, transit schedule
131 changes, special events, road improvements, land use changes, etc. When the
132 expected average crash frequency of a roadway or facility is estimated using the

¹ ' α ' or the level of statistical significance is the probability of reaching an incorrect conclusion, that is, of rejecting the hypothesis 'no difference' when the two expected values were actually the same

133 average of the last 'n' periods of accident counts, the estimate is of the average over
 134 these 'n' periods; it is not the estimate of the last period or some recent period. If the
 135 period-to-period differences are negligible, then the average over 'n' periods will be
 136 similar in each of the 'n' periods. However, if the period-to-period differences are not
 137 negligible, then the average over 'n' periods is not a good estimate of any specific
 138 period.

139 **Estimating average crash frequency assuming similar crash frequency in all**
 140 **periods**

141 Using the example in Exhibit A-1, the estimate for Year 4 is sought. Using only
 142 the accident count for Year 4:

- 143 ▪ The estimate is $\hat{\mu}_{year4} = 9$ accidents, and
- 144 ▪ The standard error of the estimate is $\hat{\sigma} = \sqrt{9} = \pm 3$ accidents.

145 Alternatively, using the average of all four accident counts:

- 146 ▪ The estimate is $\hat{\mu}_{year4} = (5+7+11+9)/4 = 8.0$ accidents, and
- 147 ▪ The standard error of the estimate is $\hat{\sigma} = \sqrt{32/4^{(2)}} = \pm 1.4$ accidents.

148 These results show that using the average of accident counts from all four years
 149 reduces the standard error of the estimate. However, the quality of the estimate was,
 150 in this case, not improved because the expected frequency is 10.3 accidents in Year 4,
 151 and the estimate of 9 accidents is closer than the estimate of 8.0 accidents. In this
 152 specific case, using more accident counts did not result in a better estimate of the
 153 expected crash frequency in the fourth year because the accident counts during the
 154 last year are not similar to the crash frequency in the three preceding years.

155 **Estimating average crash frequency without assuming similar crash frequency**
 156 **in all periods**

157 This estimation of the average crash frequency of a specific roadway or facility in
 158 a certain period is conducted using accident counts from other periods without
 159 assuming that the expected average crash frequency of a specific roadway or facility's
 160 expected average crash frequency is similar in all periods. Equation A-5 presents the
 161 relationship that estimates a specific unit for the last period of a sequence.

$$162 \quad \hat{\mu}_y = \frac{\sum_{y=1}^Y X_y}{\sum_{y=1}^Y d_y} \quad (A-5)$$

163 Where,

164 $\hat{\mu}_y$ = most likely estimate of μ_Y (last period or year);

165 $\mu_y \equiv \mu_Y \times d_y$ where y denotes a period or a year ($y=1, 2, \dots, Y$;
 166 while Y denotes the last period or last year); e.g., for first
 167 period $d_1 = \text{relationship of } \mu_1 / \mu_Y$.

168 X_y = the counts of accidents for each period or year y.

169 Equation A-6 presents the estimate of the variance of $\hat{\mu}_y$.

$$\hat{V}(\hat{\mu}_y) = \sum_{y=1}^Y X_y / \left(\sum_{y=1}^Y d_y \right)^{(2)} \tag{A-6}$$

171 Where,

172 $\hat{\mu}_y$ = most likely estimate of μ_Y (last period or year);

173 d_y = the μ_1/μ_y

174 X_y = the counts of accidents for each period or year y.

175 For this estimate, it is necessary to add all accident counts reported during this
 176 year for all intersections that are similar to the intersection, under evaluation,
 177 throughout the network. Using the example given in Exhibit A-1 to illustrate this
 178 estimate, the proportion of the accidents counts per year in relation to the annual
 179 total accident counts for all similar intersections was calculated. The results are
 180 shown in Exhibit A-3, e.g. 27% of annual accidents occur in the first year, 22% in the
 181 second year, etc.

182 Each yearly proportion is modified in relation to the last year, e.g. $d_1 = \mu_1/\mu_4 =$
 183 $0.27/0.31=0.87$, as shown in Exhibit A-3.

184 **Exhibit A-3: Illustration of Yearly Proportions and Relative Last Year Rates**

	Year 1	Year 2	Year 3	Year 4=Y
Proportion of Accidents	0.27	0.22	0.20	0.31
d_y (relative to the last year)	0.87	0.71	0.64	1

185

186 For each year, the accidents counts are 5, 7, 11, and 9, see Exhibit A-1. Using
 187 Equation A-5 and Equation A-6:

188 $\hat{\mu}_{year\ 4} = (5+7+11+9) / (0.87+0.71+0.64+1) = 32/3.22 = 9.94$ estimate of accidents
 189 for the last year:

190 $\hat{\sigma} = \sqrt{32/3.22^2} = \pm 1.8$ accidents as the standard error of the last year's estimate

191 This method eliminates the need to restrict the data to recent counts and results
 192 in increased reliability by using all relevant accident counts. This method also results
 193 in a more defensible estimate because the use of d_y allows for change over the period
 194 from which accident counts are used.

195 **Estimating average crash frequency using the longer accident record history**

196 The estimate shown below uses historical traffic volumes (Annual Average Daily
 197 Traffic or AADT) and historical accident counts. The reliability of the estimate is
 198 expected to increase with the number of years used.

199 This example is shown in Exhibit A-4 where nine years (Row 1) of accident
 200 counts (Row 4) and AADT volumes (Row 3) for a one-mile of road are presented. The
 201 estimate of the expected annual crash frequency is needed for this road segment in
 202 1997, the most recent year of data entry.

203 For this road type, the safety performance function (SPFs are discussed in Section
 204 3.5.1.) showed that the expected average crash frequency changes in proportion to
 205 AADT as shown in Equation A-7

206
$$d_y = (AADT_y / AADT_n)^{0.8} \tag{A-7}$$

207 Where,

208 $AADT_y$ = average daily traffic volume for each year y;

209 $AADT_n$ = average daily traffic volume for last year y

210 For example, the corresponding value of $d_{5=1993} = (5600/5400)^{0.8} = 1.030$.

211 The $\mu_{Y=1997}$ estimate of expected accidents would be 6.00±2.45 accidents when
 212 using Equation A-5 and Equation A-6 and the accident count for 1997 only. The
 213 $\mu_{Y=1997}$ estimate of expected accidents would be 6.09±1.44 accidents when using
 214 Equation A-5 and Equation A-6 and the accident counts for 1995, 1996 and 1997.

215 **Exhibit A-4: Estimates of Expected Average Crash Frequency Using the Longer Accident**
 216 **History**

Data										
1	Year	1989	1990	1991	1992	1993	1994	1995	1996	1997
2	Y	1	2	3	4	5	6	7	8	Y=9
3	AADT	4500	4700	5100	5200	5600	5400	5300	5200	5400
4	Accidents, X_y	12	5	9	8	14	8	5	7	6
Computations										
5	$d_y = (AADT_y / AADT_{1997})^{0.8}$	0.864	0.895	0.955	0.970	1.030	1.000	0.985	0.970	1.000
6	Cumulative Accidents	74	62	57	48	40	26	18	13	6
7	Cumulative d_y	8.670	7.805	6.910	5.955	4.985	3.955	2.955	1.970	1.000
8	Estimates of μ_{1997}	8.54	7.94	8.25	8.06	8.02	6.57	6.09	6.60	6.00
9	Standard errors	0.99	1.01	1.09	1.16	1.27	1.29	1.44	1.83	2.45
10	No. of years used	9	8	7	6	5	4	3	2	1

217
 218 This example shows that when the estimate μ_Y is based on one single accident
 219 count X_Y , no assumptions need to be made, but the estimate is inaccurate (the
 220 standard error is 2.45). When accident counts of other years are used to increase
 221 estimation reliability (the standard error decreases with the additional years of data
 222 to a value of 0.99 when adding all nine years), some assumption always needs to be
 223 made. It is assumed that the additional years from which the accident counts are
 224 used have the same estimate μ as year Y (last year).

225 **A.4 Estimating Average Crash Frequency**
 226 **Based on Historic Data of Similar**
 227 **Roadways or Facilities**

228 This section shows how the crash frequency of a specific roadway, facility or unit
 229 can be estimated using information from a group of similar roadways or facilities.
 230 This approach is especially necessary when accidents are very rare, such as at rail-
 231 highway grade crossings where accidents occur on average once in 50 years and
 232 when the accident counts of a roadway or facility cannot lead to useful estimates. The
 233 two key ideas are that:

- 234 1. Roadways or facilities similar in some, but not all, attributes will have a
 235 different expected number of accidents (μ 's) and this can be described by a
 236 statistical function called the 'probability density function.' The $E\{\mu\}$ and
 237 $V\{\mu\}$ are the mean and the variance of the group (represented by the
 238 function), and $\hat{E}\{\mu\}$ and $\hat{\sigma}_v^2\{\mu\}$ are the estimates of the expected average
 239 crash frequency and the variance.
- 240 2. The specific roadway or facility for which the estimate forms part of the
 241 group (the population of similar roadways or facilities) in a formal way. The
 242 best estimate of its estimate μ , the expected number of accidents, is $\hat{E}\{\mu\}$ and
 243 the standard error of this estimate is $\hat{V}\{\mu\}$, both of which are derived from
 244 the estimates of the group's function.

245 In practice, as groupings of similar roadways or facilities are only samples of the
 246 population of such roadways or facilities, the estimates of the mean and variances of
 247 the probability density function will be based on the sample of similar roadways or
 248 facilities. The estimates use Equation A-8 and Equation A-9.

$$249 \quad \bar{x} = \sum_1^n \left(\frac{x_i}{n} \right) \quad (A-8)$$

250 Where,

251 \bar{x} = mean of accident counts for the group or sample of similar
 252 roadways or facilities;

253 x_i (i=1,2,...n) = accident counts for n roadways or facilities similar to the
 254 roadway or facility of which crash frequency is estimated.

$$255 \quad s^2 = \sum_1^n \frac{(x_i - \bar{x})^2}{(n-1)} \quad (A-9)$$

256 Where,

257 s^2 = variance of accident counts for the group or sample of similar
 258 roadways or facilities;

259 x_i (i=1,2,...n) = accident counts for n roadways or facilities similar to the
 260 roadway or facility of which crash frequency is estimated.

261

262

263

264

265 The estimate of the crash frequency of a specific roadway, facility or unit is
 266 calculated by using Equation A-10.

$$267 \quad \hat{E}\{\mu\} = \bar{x} \text{ and } \hat{V}\{\mu\} = s^2 - \bar{x} \quad (A-10)$$

268 Where,

269 $\hat{E}\{\mu\}$ = expected number of accidents for a roadway or facility based
 270 on the group of similar roadways or facilities;

- 271 \bar{X} = mean of accident counts for the group or sample of similar
- 272 roadways or facilities;
- 273 $\hat{V}\{\mu\}$ = variance for the expected number of accidents for a roadway
- 274 or facility based on the group of similar roadways or
- 275 facilities;
- 276 S^2 = variance of accident counts for the group or sample of similar
- 277 roadways or facilities.

278 Exhibit A-5 provides an example that illustrates the application of historic data
 279 from similar facilities. This example estimates the expected average crash frequency
 280 of a rail-highway at-grade crossing in Chicago for 2004. The crossing in Chicago has
 281 one rail track, 2 trains per day, and 500 vehicles per day. The crossing is equipped
 282 with crossbucks.

283 As the accident history of this crossing is not sufficient (small sample size) for the
 284 estimation of its expected average crash frequency, the estimate uses national
 285 accident historical data for rail-highway crossings. Exhibit A-5 sets out accident data
 286 for urban rail-highway at-grade crossings in the United States for crossings that have
 287 similar attributes to the crossing in Chicago⁽⁴⁾.

288 **Exhibit A-5: National Accident Data for Railroad-Highway Grade Crossings (with 0-1,000**
 289 **vehicles/day, 1-2 trains/day, single track, urban area) 2004**

Number of Accident Counts/Year(2004) (x_i)	Number of Crossings (n_i)	$A_j = (x_i) \times (n_i) / N$	$S_j = [(x_i) - \bar{X}]^2 \times (n_i) / (N-1)$
0	10234	0.0000	0.0003
1	160	0.0154	0.0148
2	11	0.0021	0.0042
3	3	0.0009	0.0026
	$\sum n = N$ =10408 total similar crossings	$\bar{X} = \sum A_j =$ = 0.0184 expected accidents /year per crossing in this group	$s^2 = \sum S_j =$ 0.0219

290

291 Using Equation A-10 and the data shown for similar crossings in Exhibit A-5, a
 292 reasonable estimate of the crash frequency of the crossing in Chicago for 2004 is
 293 0.0184 accidents/year, i.e., the same as the sample mean (\bar{X}). The standard error is
 294 estimated as $\sqrt{0.0219 - 0.0184} = \pm 0.059$ accidents/year.

295 It was possible to calculate this estimate because rail-highway at-grade crossings
 296 are numerous and official statistics about the crossings are available.

297 For roadways or facilities such as road segments, intersections, and interchanges,
 298 it is not possible to obtain data from a sufficient number of roadways or facilities
 299 with similar attributes. In these circumstances, SPFs and other multivariable
 300 regression models (Part III) are used to estimate the mean of the probability
 301 distribution and its standard error. Section A.5 describes the use of SPFs to improve
 302 the estimation of the expected average crash frequency of a facility.

A.5 Estimating Average Crash Frequency Based on Historic Data of the Roadway or Facilities and Similar Roadways and Facilities

The estimation of expected average crash frequency of a certain roadway or facility can be improved, i.e., the reliability of the estimate can be increased, by combining the roadway or facility's count of past accidents (Section A.3) with the accident record of similar roadways or facilities (Section A.4).

The "best" estimate combined with the minimum variance or standard error is given by Equation A-11.

$$\hat{\mu} = \omega \times \hat{\mu}_s + (1 - \omega) \times \hat{\mu}_a \quad (A-11)$$

where

$$\omega = \frac{1}{\left(1 + \frac{V\{\mu_s\}}{E\{\mu_s\}}\right)}$$

Where,

$\hat{\mu}$ = the "best" estimate of a given roadway or facility;

$\hat{\mu}_s$ = the estimate based on data of a group of similar roadways or facilities;

$\hat{\mu}_a$ = the estimate based on accident counts of the given roadway or facility;

$V\{\mu_s\}$ = variance of the estimate based on data for similar roadways or facilities;

$E\{\mu_s\}$ = the estimate of expected average crash frequency based on the group of similar roadways or facilities;

ω = the weight based on the estimate and the degree of its variance resulting from the grouping of similar roadways or facilities.

When $\hat{\mu}$ is estimated by Equation A-11, its variance is given by Equation A-12.

$$V(\hat{\mu}) = \omega \times V\{\mu_s\} = (1 - \omega) \times E\{\mu_s\} \quad (A-12)$$

Where,

$V(\hat{\mu})$ = variance of the "best" estimate;

$V\{\mu_s\}$ = variance of the estimate based on data from similar units or a group of similar roadways or facilities;

$E\{\mu_s\}$ = the estimate of expected number of accidents based on the group of similar roadways or facilities;

335 ω = weight generated by the variance of the estimate of expected
336 average crash frequency.

337 As an example, the expected average crash frequency of a 1.23 mile section of a
338 six-lane urban freeway in Colorado is estimated below. The estimate is based on 76
339 accidents reported during a 3-year period, and accident data for similar sections of
340 urban freeways.

341 There are 3 steps in the estimation:

342 **Step 1: As expressed by Equation A-3, using the accidents reported for the**
343 **specific roadway or facility:**

$$344 \quad \hat{\mu}_i = x = 76 \text{ accidents} \quad \text{and} \quad \hat{\sigma}_i = \sqrt{x} = \pm 8.7 \text{ accidents} \quad (A-3)$$

345 Where,

346 $\hat{\mu}_i$ = the expected number of accidents for a roadway or facility for
347 period i ;

348 x = the reported number of accidents for this roadway or facility
349 and period i;

350 $\hat{\sigma}_i$ = standard error for the expected number of accidents for this
351 roadway or facility and period i.

352 **Step 2: Based on AADT volumes, the percentage of trucks, and accident counts**
353 **on similar urban freeways in Colorado, a multivariable regression model was**
354 **calibrated (Section B.1). When the model was applied to a 1.23 mile section for**
355 **a 3-year period, the following estimates (Equation A-10) result:**

$$356 \quad \hat{E}\{\mu_s\} = \hat{E}\{\mu\} = \bar{x} = 61.3 \text{ accidents}$$

$$357 \quad \hat{V}\{\mu_s\} = \hat{V}\{\mu\} = s^2 - \bar{x} = 266.7 \text{ accidents}^2$$

$$358 \quad \hat{\sigma}_i = \sqrt{s^2 - \bar{x}} = \pm 16.3 \text{ accidents}$$

359 Where,

360 $\hat{E}\{\mu_s\}$ = the estimate of expected number of accidents based on the
361 group of similar roadways or facilities;

362 $\hat{V}\{\mu_s\}$ = the estimate of the variance of $\hat{E}\{\mu_s\}$;

363 \bar{x} = mean of accident counts for the group of similar roadways or
364 facilities for the AADT volume and truck percentage for the
365 specific roadway or facility;

366 $\hat{V}\{\mu\}$ = variance for the expected number of accidents for the specific
367 roadway or facility based on the group's model;

368 s^2 = variance of accident counts for the group or sample of similar
369 roadways or facilities ;

370 $\hat{\sigma}_i$ = standard error for the expected number of accidents for the
371 specific roadway or facility based on the group's model.

372 **Step 3: using the statistical relative weight of the two estimates obtained from**
 373 **Step 1 and Step 2, the 'best' estimate of the expected number of accidents on**
 374 **this 1.23 mile of urban freeway is:**

375 The 'weight' ω (Equation A-11) is:

376
$$\omega = \frac{1}{\left(1 + \frac{V\{\mu_s\}}{E\{\mu_s\}}\right)} \tag{A-11}$$

377 Where,

378 $V\{\mu_s\}$ = variance of the estimate based on data about similar units or
 379 groups;

380 $E\{\mu_s\}$ = the estimate of expected number of accidents based on the
 381 group of similar roadways or facilities;

382 Thus:

383
$$\omega = 1 / (1 + 266.7 / 61.3) = 0.187$$

384 The "best" estimate of a given unit, roadway or facility is estimated as:

385
$$\hat{\mu} = \omega \times \hat{\mu}_s + (1 - \omega) \times \hat{\mu}_a \tag{A-11}$$

386 with the variance as:

387
$$V(\hat{\mu}) = \omega \times V\{\mu_s\} = (1 - \omega) \times E\{\mu_s\} \tag{A-12}$$

388 Where,

389 $\hat{\mu}$ = the "best" estimate of a certain roadway or facility;

390 $\hat{\mu}_s$ = the estimate based on data about similar units or group of
 391 similar roadways or facilities;

392 $\hat{\mu}_a$ = the estimate based on accident counts;

393 ω = the weight indicative of the estimate and the degree of its
 394 variance resulting from the grouping of similar roadways or
 395 facilities;

396 $\hat{V}\{\mu\}$ = variance for the expected average crash frequency for a
 397 certain roadway or facility based on the group's model;

398 $\hat{E}\{\mu_s\}$ = the estimate of expected average crash frequency based on
 399 the group of similar roadways or facilities;

400 $\hat{V}\{\mu_s\}$ = the estimate of the variance of $\hat{E}\{\mu_s\}$

401 Thus:

402
$$V\{\hat{\mu}\} = (1 - 0.187) \times 61.3 = 49.83 \text{ accidents}^2$$

403
$$\hat{\sigma}_i = \pm 7.1 \text{ accidents}$$

404 Exhibit A-6 shows the results of the three steps, and that the estimate that
 405 combines the estimation of a certain roadway or facility with the estimation of similar
 406 roadways or facilities results in an estimation with the smallest standard of error.

407 **Exhibit A-6: Comparison of Three Estimates (an example using accident counts, groups**
 408 **of similar roadways or facilities, and combination of both)**

	Expected Number of Accidents (3 years)	Standard Error
Estimate based only on accident counts	76.0	± 8.7
Estimate based only on data about similar roadways or facilities	61.3	± 16.3
Estimate based on both accident counts and data about similar roadways or facilities	73.3	± 7.1

409
 410 Another example that illustrates the use of a SPF in the estimation of the
 411 expected average crash frequency of a facility is shown below. SPFs were derived for
 412 stop-controlled and signalized four-leg intersections.^(15,17) The chosen function for
 413 both types of intersection control is shown in Equation A-13.

414
$$\hat{E}\{\mu\} = a \times F_{Major}^{(\beta_1)} \times F_{Minor}^{(\beta_2)} \times e^{(\beta_3 F_{Minor})} \quad (A-13)$$

- 415 Where,
- 416 $\hat{E}\{\mu\}$ = the estimate of the average expected frequency of injury
 417 accidents;
- 418 F = the entering AADT on the major and minor approaches;
- 419 α, β_1, β_2 and β_3 = the estimated constants shown in Exhibit A-7;
- 420 e = base of natural logarithm function.

421
422
423

Exhibit A-7: Estimated Constants for Stop-Controlled and Signalized Four-Leg Intersections SPF Shown in Equation A-13 Including the Statistical Parameter of Overdispersion ϕ (an example)

	Stop-controlled ⁽¹⁷⁾	Signalized ⁽¹⁵⁾
α	3.22×10^{-4}	8.2×10^{-5}
β_1	0.50	0.57
β_2	0.43	0.55
β_3	0 (not in model)	6.04×10^{-6}
ϕ	2.3	4.6

424

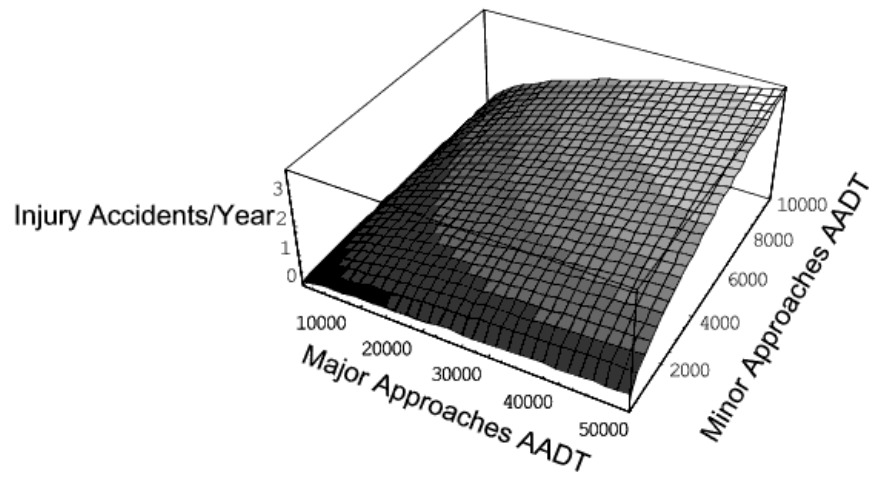
425

426

The surfaces of the two SPFs (one for stop-controlled intersections and one for signalized four-leg intersections) are shown in Exhibit A-8 and Exhibit A-9.

427

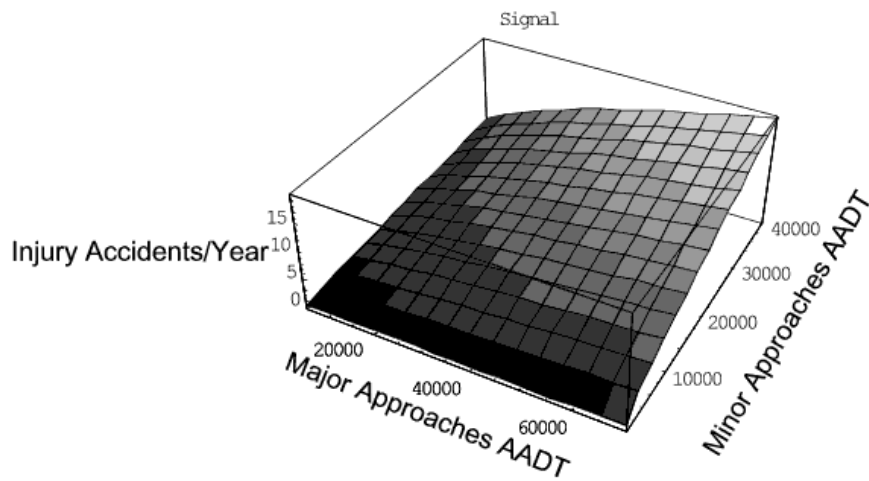
Exhibit A-8: Estimated Injury Accidents at Stop-Controlled Four-Leg Intersections



428

429

430 **Exhibit A-9: Predicted Injury Accidents at Signalized Four-Leg Intersections**



431
 432 AADT is a major attribute when considering crash frequency, but there are many
 433 other attributes which, although not explicitly shown in the SPF, influence the
 434 estimate for a given facility or roadway. In the example above, many attributes of the
 435 two groups of intersections, besides AADT, contribute to the values for $E\{\mu\}$
 436 computed Equation A-17 for major and minor approach AADTs. Inevitably, the
 437 difference between any two values is an approximation of the change expected if,
 438 for example, a stop-controlled intersection is signalized, because it does not separate the
 439 many attributes other than traffic control device.

440

441

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APPENDIX B—DERIVATION OF SPFS

447 The variables and terminology presented in this appendix are not always
448 consistent with the material in Chapter 3.

449

B.1 Safety Performance as a Regression Function

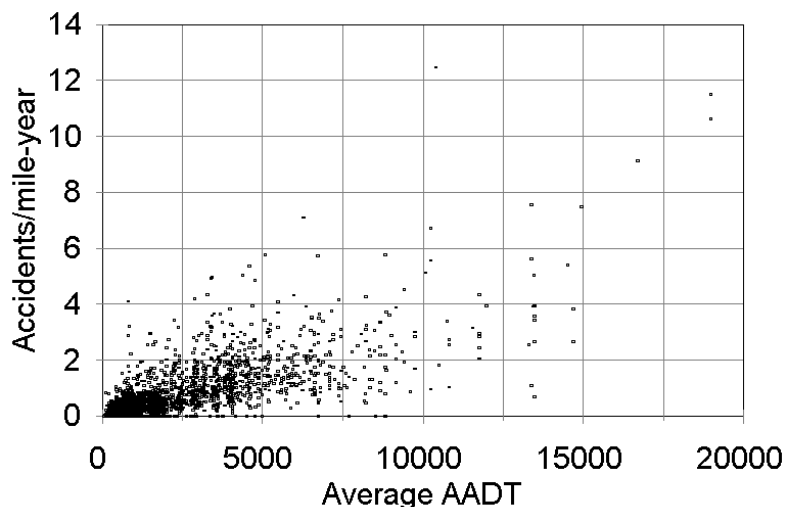
450

451 SPFs are developed through statistical regression modeling using historic
452 accident data collected over a number of years at sites with similar roadway
453 characteristics. The validity of this process is illustrated conceptually though the
454 following example using Colorado data for rural two-lane road segments (excluding
455 intersections). Segment length, terrain type (mountainous or rolling), crash frequency
456 and traffic volumes were collected for each year from 1986 to 1998. Crashes per mile-
457 year for each site were plotted against traffic volume, based on average AADT over
458 the 13-year period. The data points were then separated by terrain type to account for
459 the different environmental factors of each type. The crash frequency plot for rural
460 two-lane roads with rolling terrain is shown in Exhibit B-1.

461

Exhibit B-1: Crashes per Mile-Year by AADT for Colorado Rural Two-Lane Roads in Rolling Terrain (1986-1998)

462

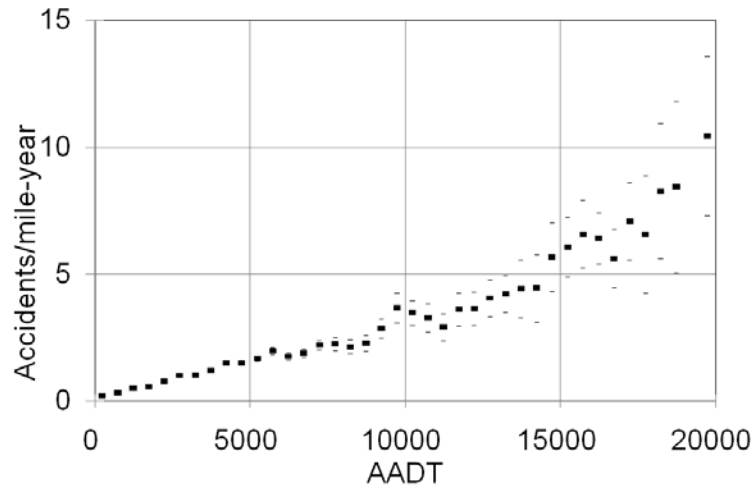


463

464 The variability in the points in the plot reflects the randomness in crash
465 frequency, the uncertainty of AADT estimates, and characteristics that would affect
466 expected average crash frequency but were not fully accounted for in this analysis,
467 such as grade, alignment, percent trucks, and number of driveways. Despite the
468 variability of the points, it is still possible to develop a relationship between expected
469 average crash frequency and AADT by averaging the number of crashes. Exhibit B-2
470 shows the results of grouping the crashes into AADT bins of 500 vehicles/day, that
471 is, averaging the number of crashes for all points within a 500 vehicles/day
472 increment.

473
474

Exhibit B-2: Grouped Crashes per Mile-Year by AADT for Colorado Rural Two-Lane Roads in Rolling Terrain (1986-1998)



475

NOTE: The black squares are the ratio of the number of accidents for all road sections in a bin divided by the sum of the corresponding road segment lengths. The bars around the black squares are ± two standard errors of this ratio.

476
477
478

479 Exhibit B-2 illustrates that in this case, there is a relationship between accidents
480 and AADT, when using average bins. These associations can be captured by
481 continuous functions which are fitted to the original data. The advantage of fitting a
482 continuous function is to smooth out the randomness where data are sparse, such as
483 for AADTs greater than 15,000 vehicles/day in this example. Based on the regression
484 analysis, the “best fit” SPF for rural two-lane roads with rolling terrain from this
485 example is shown in Equation B-1. *Note that this is not the SPF for rural two-lane*
486 *two-way roads presented in Chapter 10 of the HSM. As the base conditions of the*
487 *SPF model shown below are not provided, its use is not recommended for application*
488 *with the Part C predictive method.*

489

$$\hat{E}\{\mu\} = 1.95 \times \left(\frac{AADT}{1000}\right)^{0.71} \times e^{\left(0.53 \times \left(\frac{AADT}{1000}\right)\right)} \quad (B-1)$$

490

Where,

491

$\hat{E}\{\mu\}$ = the estimate of the average crash frequency per mile;

492

AADT = the average annual daily traffic.

493

The overdispersion parameter for rural two-lane roads with rolling terrain in Colorado from this example was found to be 4.81 per mile.

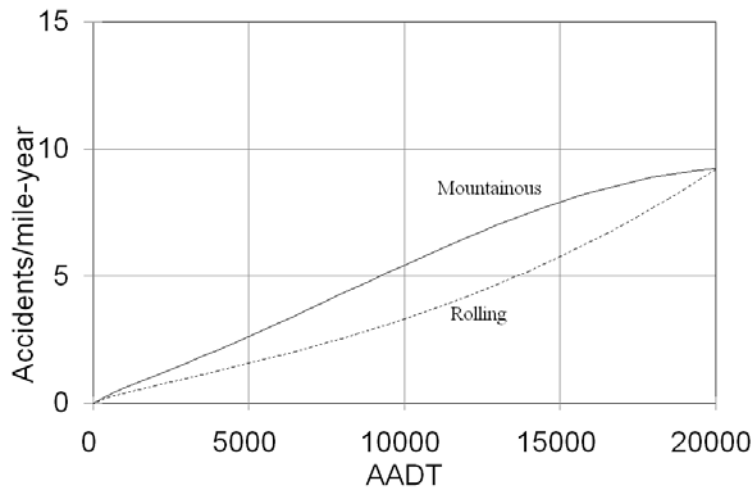
494

495

The SPF for rural two-lane roadways on rolling terrain shown in Equation B-1 is depicted in Exhibit B-3 alongside a similar SPF derived for mountainous terrain.

496

497 **Exhibit B-3: Safety Performance Functions for Rural Two-Lane Roads by Terrain Type**



498

499 **B.2 Using a Safety Performance Function to**
 500 **Predict and Estimate Average Crash**
 501 **Frequency**

502 Using the SPFs shown in Exhibit B-3, an average two-lane rural road in Colorado
 503 with AADT=10,000 vehicles/day is expected to have 3.3 accidents/mile-year if in
 504 rolling terrain and 5.4 accidents/mile-year if in mountainous terrain.

505 When an equation is fitted to data, it is also possible to estimate the variance of
 506 the expected number of accidents around the average number of accidents. This
 507 relationship is shown in Equation B-2.

508
$$V\{\mu\} = \frac{(E\{\mu\})^2}{k} \tag{B-2}$$

509 Where,

510 k = the overdispersion parameter

511 $E\{\mu\}$ = the average crash frequency per mile

512 $V\{\mu\}$ = the variance of the average crash frequency per mile

513 As an example to illustrate its use, Exhibit B-3 shows that an *average* two-lane
 514 rural road in a rolling terrain in Colorado with AADT=10,000 vehicles/day is
 515 expected to have 3.3 accidents/mile-year. Thus, for a road segment with 0.27mile
 516 length, it is expected that there will be on average $0.27 \times 3.3 = 0.89$ accidents/year.

517 When the SPF for two-lane roads in Colorado was developed, the overdispersion
 518 parameter (k) for rolling terrain was found to be 4.81/mile.

519 Thus:

520 $\hat{V}\{\mu\} = \text{variance} = (E\{\mu\})^2 / \phi = 0.89^2 / (0.27 \times 4.81)$

521 $= 0.55 \text{ (accidents/year)}^2 \text{ or}$

522 $\hat{\sigma}\{\mu\} = \text{standard error} = \sqrt{0.55} = \pm 0.74 \text{ accidents/year}$

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524
525

APPENDIX C—AMF AND STANDARD ERROR

526
527

528 The variables and terminology presented in this appendix are not always
529 consistent with the material in Chapter 3.

530 The more precise an AMF estimate, the smaller its standard error. The reliability
531 level of AMFs is illustrated by means of probability density functions. A probability
532 density function is any function $f(x)$ that describes the probability density in terms of
533 the input variable x in the manner described below:

- 534 ■ $f(x)$ is greater than or equal to zero for all values of x
- 535 ■ The total area under the graph is 1:

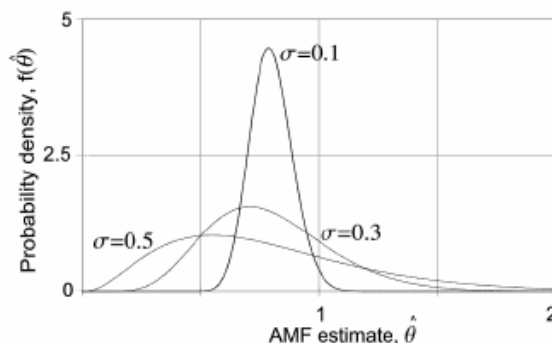
$$536 \int_{-\infty}^{\infty} f(x)dx = 1 \quad (C-1)$$

537 In other words, a probability density function can be seen as a “smoothed out”
538 version of the histogram that one would obtain if one could empirically sample
539 enough values of a continuous random variable.

540 Different studies have different probability density functions, depending on such
541 factors as the size of the sample used in the study and the quality of the study design.
542 Exhibit C-1 shows three alternative probability density functions of an AMF estimate.
543 These functions have different shapes with different estimates of AMFs at the peak
544 point, i.e. at the mode (the most frequent value) of the function. The mean value of all
545 three probability density functions is 0.8. The value of the standard error indicates
546 three key pieces of information:

- 547 1. The compact probability density function with standard error $\sigma = 0.1$
548 represents the results of an evaluation research study using a fairly large
549 data set and good method
- 550 2. The probability density function with standard error $\sigma = 0.3$ represents the
551 results of a study that is intermediate between a good and a weak study
- 552 3. The wide probability density function with standard error $\sigma = 0.5$ represents
553 the results of a study that is weak in data and/or method

554 **Exhibit C-1: Three Alternative Probability Density Functions of AMF Estimates**



555
556
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558

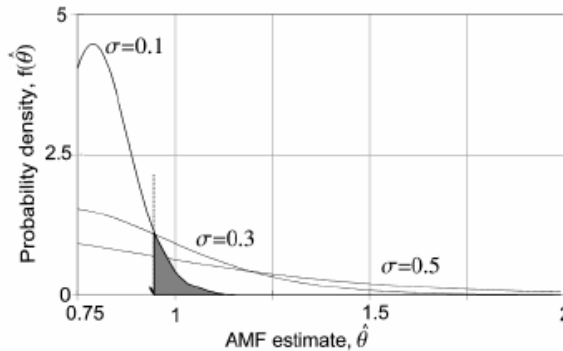
As an example of the use of AMFs and standard errors, consider a non-expensive
and easy to install treatment that might or might not be implemented. The cost of this
installation can be justified if the expected reduction in accidents is at least 5% (i.e., if

559 $\theta < 0.95$). Using the AMF estimates in Exhibit C-1 for this particular case, if the AMF
 560 estimate is 0.80 (true and mean value of θ , as shown in Exhibit C-1), the reduction in
 561 expected accidents is clearly greater than 5% ($\theta = 0.8 < 0.95$).

562 However, the key question is: ‘what is the chance that installing this treatment is
 563 the wrong decision?’ Whether the AMF estimate comes from the good, intermediate,
 564 or weak study, will define the confidence in the decision to implement.

565 The probability of making the wrong decision by accepting an AMF estimate
 566 from the good study ($\sigma = 0.1$ in Exhibit C-1) is 6%, as shown by the shaded area in
 567 Exhibit C-2 (the area under the graph to the right of the 0.95 estimate point). If the
 568 AMF estimate came from the intermediate study ($\sigma = 0.3$ in Exhibit C-1), the
 569 probability of making an incorrect decision is about 27%. If the AMF estimate came
 570 from the weak study ($\sigma = 0.5$ in Exhibit C-1) the probability of making an incorrect
 571 decision is more than 31%.

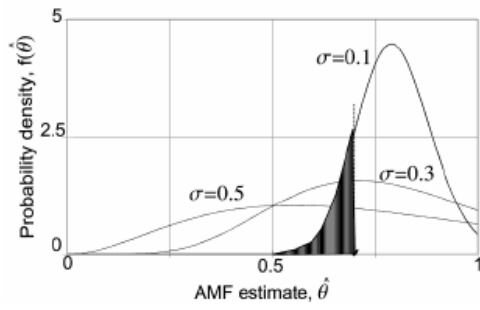
572 **Exhibit C-2: The Right Portion of Exhibit C-1; Implement if AMF < 0.95**



573
 574 Likewise, what is the chance of making the wrong decision about installing a
 575 treatment that is expensive and not easy to implement, and that can be justified only
 576 if the expected reduction in accidents is at least 30% (i.e., if $\theta < 0.70$). Using the AMF
 577 estimates in Exhibit C-1 for this particular case, implementing this intervention
 578 would be an incorrect decision because $\theta = 0.80$ (Exhibit C-1) is larger than the $\theta =$
 579 0.70 which is required to justify the installation cost.

580 The probability of making the wrong decision by accepting an AMF estimate
 581 from the good study ($\sigma = 0.1$ in Exhibit C-1) is 12%, as shown by the shaded area in
 582 Exhibit C-3 (the area under the graph to the left of the 0.70 estimate point). If the
 583 AMF estimate came from the intermediate study ($\sigma = 0.3$ in Exhibit C-1), the
 584 probability of making an incorrect decision is about 38%. If the AMF estimate came
 585 from the weak study ($\sigma = 0.5$ in Exhibit C-1) the probability of making an incorrect
 586 decision is about 48%.

587 **Exhibit C-3: The Left Portion of Exhibit C-1; Implement if AMF < 0.70**



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593

APPENDIX D—INDIRECT SAFETY MEASUREMENT

594
595

596 The variables and terminology presented in this appendix are not always
597 consistent with the material in Chapter 3.

598 Indirect safety measurements, also known as safety surrogate measures, were
599 introduced in Section 3.4 and are described in further detail here. They provide the
600 opportunity to assess safety when accident counts are not available because the
601 roadway or facility is not yet in service or has only been in service for a short time, or
602 when crash counts are few or have not been collected, or when a roadway or facility
603 has significant unique features. The important added attraction of indirect safety
604 measurements is that they may save having to wait for sufficient accidents to
605 materialize before a problem is recognized and the remedy applied. In addition,
606 knowledge of the pattern of events that precedes accidents might provide an
607 indication of appropriate preventative measures. The relationships between potential
608 surrogate measures and expected crashes have been studied and are discussed
609 below.

610 *The Heinrich Triangle and Two Basic Types of Surrogates*

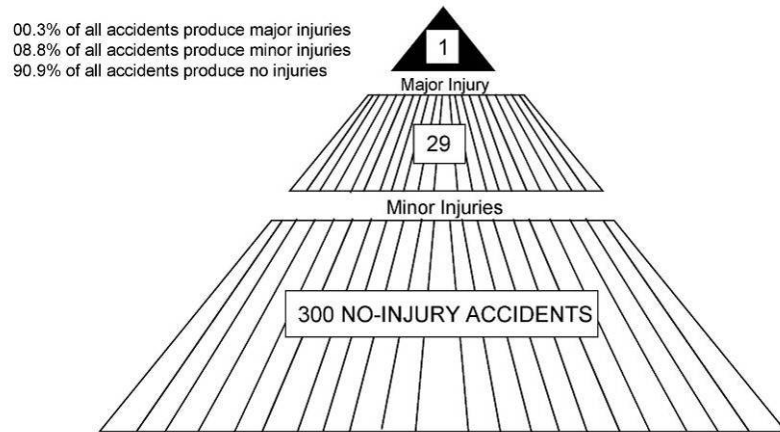
611 Past practices have mostly used two basic types of surrogate measures. These
612 are:

- 613 ■ Surrogates based on events which are proximate to and usually precede the
614 accident event
- 615 ■ Surrogates that presume existence of a causal link to expected average crash
616 accident frequency. These surrogates assume knowledge of the degree to
617 which safety is expected to change when the surrogate measure changes by a
618 given amount

619 The difference between these two types of surrogates is best explained with
620 reference to Exhibit D-1 which shows the 'Heinrich Triangle.' The 'Heinrich Triangle'
621 has set the agenda for Industrial and Occupational Safety ever since it was first
622 published in 1932.⁽¹³⁾ The original Heinrich Triangle is founded on the precedence
623 relationship that 'No Injury Accidents' precedes 'Minor Injuries.'

624

Exhibit D-1: The Heinrich Triangle



Source: H. W. Heinrich, *Industrial Accident Prevention*, 1950, p. 24.

625

626

627

There are two basic ideas:

628

- Events of lesser severity are more numerous than more severe events, and events closer to the base of the triangle precede events nearer the top

629

630

- Events near the base of the triangle occur more frequently than events near the triangle's top, and their rate of occurrence can be more reliably estimated

631

632

Events Closer to the Base of the Triangle Precede Events Nearer the Top

633

The shortest Time to Collision (TTC) illustrates the idea that events closer to the base of the triangle precede events nearer the top. The shortest TTC was proposed as a safety surrogate by Hayward in 1972⁽²¹⁾ and applied by van der Horst.⁽²²⁾ The approach involves collecting the number of events in which the $TTC \leq 1s$: events that were never less than, and are usually larger than the number of events in which $TTC \leq 0.5s$ which are never less than, and usually larger than the number of crashes (equivalent to $TTC = 0$). Thus, for all events $TTC > 0$, the event did not result in a collision. The importance of this idea for prevention is that preventing less severe events (with greater values of TTC) is likely to reduce more severe events (with lower values of TTC).

642

643

Events Near the Base Occur More Frequently and can Be More Reliably Estimated

644

645

The second basic idea of the Heinrich Triangle is that because events near the base occur more frequently than events near its top, their rate of occurrence can be more reliably estimated. Therefore, one is able to learn about changes or differences in the rate of occurrence of the rare events by observing the changes or differences in the rate of occurrence of the less severe and more frequent events.

649

650

This relationship, in its simplest form, is shown in Equation D-1.

$$\left[\begin{array}{l} \text{Number of accidents expected} \\ \text{to occur on an entity in a certain} \\ \text{period of time} \end{array} \right] = \left[\begin{array}{l} \text{Number of surrogate events} \\ \text{occurring on the entity in} \\ \text{that period of time} \end{array} \right] \times \left[\begin{array}{l} \text{Accidents per} \\ \text{surrogate event} \\ \text{for that entity} \end{array} \right]$$

651 (D-1)

652 Equation D-1 is always developed separately for each accident type. Equation D-
653 1 can be rewritten as shown in Equation D-2.

$$654 \quad \hat{\mu} = \sum_i (\hat{C}_i \times \hat{p}_i) \quad (D-2)$$

655 Where,

656 $\hat{\mu}$ = the expected average crash frequency of a roadway or facility
657 estimated by means of surrogate events;

658 \hat{C}_i = estimate of the rate of surrogate event occurrence for the
659 roadway or facility for each severity class i. The estimate is
660 obtained by field observation, by simulation, or by analysis;

661 \hat{p}_i = estimate of the accident/surrogate-event ratios for the
662 roadway or facility for each severity class i. The estimate is
663 the product of research that uses data about the occurrence of
664 surrogate events and of accidents on a set of roadways or
665 facilities.

666 The success or failure of a surrogate measure is determined by how reliably it
667 can estimate expected accidents. This is expressed by Equation D-3.⁽¹²⁾

$$668 \quad V\{\hat{\mu}\} \cong \sum (\hat{C}_i^{(2)} \times V\{\hat{p}_i\} + \hat{p}_i^{(2)} \times V\{\hat{C}_i\}) \quad (D-3)$$

669 Where,

670 \hat{C}_i = estimate of the rate of surrogate event occurrence for the
671 roadway or facility for each severity class i. The estimate is
672 obtained by field observation, by simulation, or by analysis;

673 \hat{p}_i = estimate of the accident/surrogate-event ratios for the
674 roadway or facility for each severity class i. The estimate is
675 the product of research that uses data about the occurrence of
676 surrogate events and of accidents on a set of roadways or
677 facilities;

678 $V\{\hat{C}_i\}$ = the variance of \hat{C}_i . This depends on the method by which
679 \hat{C}_i was obtained, the duration of observations, etc;

680 $V\{\hat{p}_i\}$ = the variance of \hat{p}_i . This depends mainly on the similarity of
681 \hat{p}_i from roadway or facility to roadway and facility.

682 The choice of surrogate events will determine the size of the variance $V\{\hat{p}_i\}$. A
683 good choice will be associated with a small $V\{\hat{p}_i\}$.

684 **Some surrogate measures at intersections**

685 Exhibit D-2 list several events at intersections which have been used as safety
686 surrogates in the past.⁽⁶⁾

687 **Exhibit D-2: Surrogate Measures at Intersections**

Surrogate Measure	Description
Encroachment Time (ET)	Time duration during which the turning vehicle infringes upon the right-of-way of through vehicle.
Gap Time (GT)	Time lapse between completion of encroachment by turning vehicle and the arrival time of crossing vehicle if they continue with same speed and path.
Deceleration Rate (DR)	Rate at which through vehicle needs to decelerate to avoid accident.
Proportion of Stopping Distance (PSD)	Ratio of distance available to maneuver to the distance remaining to the projected location of accident.
Post-Encroachment Time (PET)	Time lapse between end of encroachment of turning vehicle and the time that the through vehicle actually arrives at the potential point of accident.
Initially Attempted Post-Encroachment Time (IAPT)	Time lapse between commencement of encroachment by turning vehicle plus the expected time for the through vehicle to reach the point of accident and the completion time of encroachment by turning vehicle.
Time to Collision (TTC)	Expected time for two vehicles to collide if they remain at their present speed and on the same path.

688

689 The reliability of the events listed in Exhibit D-2 in predicting expected accidents
690 has not been fully proven.

691 Other types of surrogate measures are those construed more broadly to mean
692 anything “that can be used to estimate average crash frequency and resulting
693 injuries and deaths.”⁽⁷⁾ Such surrogate measures include driver workload, mean
694 speed, speed variance, proportion of belted occupants, and number of intoxicated
695 drivers.

696 From research conducted since the ‘Heinrich Triangle’ (Exhibit D-1) was
697 developed, it is now known that for many circumstances, such as pedestrian
698 accidents to seniors, almost every accident leads to injury. For these circumstances,
699 the ‘No Injury Accidents’ layer is much narrower than the one shown in Exhibit D-1.

700 Furthermore, it is also known that, for many circumstances, preventing events of
701 lesser severity may not translate into a reduction of events of larger severity. An
702 example is the installation of a median barrier where the barrier increases the number
703 of injury accidents due to hits of the barrier, but reduces fatalities by largely
704 eliminating cross-median crashes. In the case of median barriers, the logic of Heinrich
705 Triangle’ (Exhibit D-1) does not apply because the events that lead to fatalities
706 (median crossings) are not the same events as those that lead to injuries and
707 property-damage (barrier hits).

708 In 2006, a new approach to the use of surrogates was under investigation.⁽²³⁾ This
709 approach observes and records the magnitude of surrogates such as Time-To-
710 Collision (TTC) or Post-Encroachment-Time (PET). The observed values of the
711 surrogate event are shown as a histogram for which values near 0 are missing. An
712 accident occurs when TTC or PET are 0. The study is using Extreme Value Theory to

713 estimate the missing values, thus the number of accident events implied by the
714 observed data.
715

APPENDIX E—SPEED AND SAFETY

716

717 The variables and terminology presented in this appendix are not always
718 consistent with the material in Chapter 3.

719 Driving is a self-paced task: the driver controls the speed of travel and does so
720 according to perceived and actual conditions. The driver adapts to roadway
721 conditions and adjacent land use and environment, and one of these adaptations is
722 operating speed. The relationship between speed and safety depends on human
723 behavior, and driver adaptation to roadway design, traffic control, and other
724 roadway conditions.

725 Recent studies have shown that certain roadway conditions, such as a newly
726 resurfaced roadway, result in changes to operating speeds. ⁽¹⁴⁾

727 The relationship between speed and safety can be examined during the ‘pre-
728 event’ and the ‘event’ phases of an accident. The ‘pre-event’ phase considers the
729 probability that an accident will occur, specifically how this probability depends on
730 speed. The ‘event’ phase considers the severity of an accident, specifically the
731 relationship between speed and severity. Identifying the errors that contribute to the
732 cause of crashes helps to better identify potential countermeasures.

733 The following sections describe the pre-event phase and the relationship between
734 speed and the probability of an accident (Section E.1), the event phase and the
735 relationship between the severity of an accident and change in speed at impact
736 (Section E.2), and the relationship between average operating speed and crash
737 frequency (Section E.3). In the following discussion, terms such as running speed and
738 travel speed are used interchangeably.

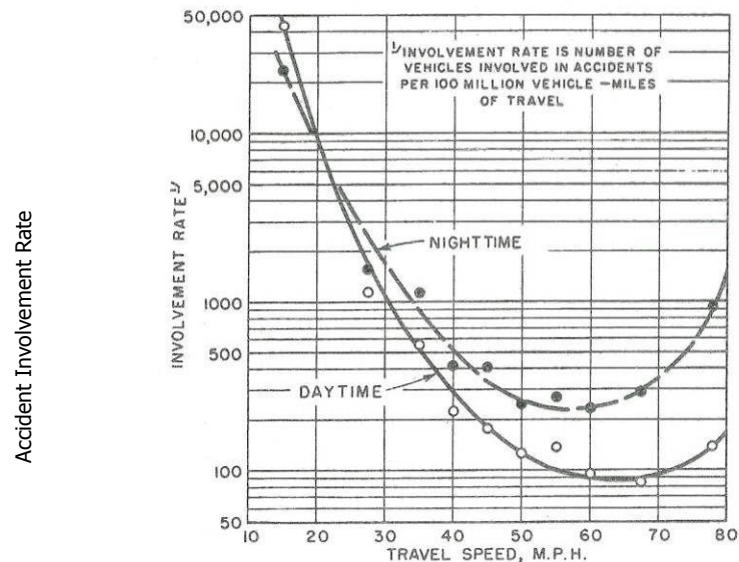
739 **E.1 Pre-Event or Pre-Crash Phase: Accident** 740 **Probability and Running Speed**

741 It is known that with higher running speeds, a longer stopping distance is
742 required. It is therefore assumed that the probability of an accident increases with
743 higher running speeds. However, while opinions on the probability of an accident
744 and speed are strongly held, empirical findings are less clear. ⁽²¹⁾

745 For example, Exhibit E-1 shows that vehicles traveling at speeds approaching
746 50 mph, are less involved in accidents than vehicles traveling at lower speeds. This is
747 the opposite of the assumed relationship between speed and accident probability in
748 terms of accident involvement rate.

749

Exhibit E-1: Accident Involvement Rate by Travel Speed ⁽²²⁾



750

751

(Reproduced from Solomon's Figure 2) ⁽²²⁾

752

753

754

755

756

The data used to create Exhibit E-1 included turning vehicles.⁽²¹⁾ Therefore accidents that appear to be related to low speeds may in fact be related to a maneuver that required a reduced speed. In addition, the shape of the curve in exhibit E-1 is also explained by the statistical representation of the data, that is, the kind of data assembled leads to a U-shaped curve.⁽⁸⁾

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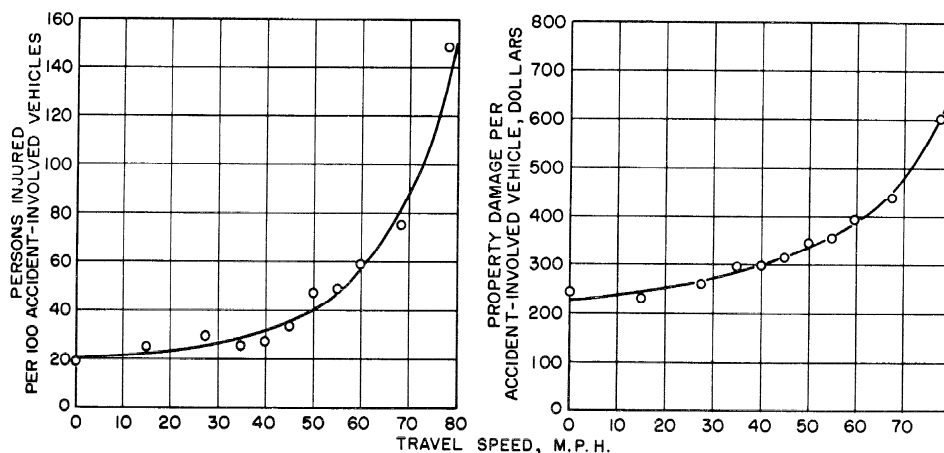
765

766

Exhibit E-1 also shows that for speeds greater than 60 mph, the probability of involvement increases with speed. At travel speeds greater than 60 mph, there is also likely to be a mixture of crash frequency and severity. Accidents of greater severity are more likely to be reported and recorded. Exhibit E-2 shows that the number of accidents by severity increases with travel speed.⁽²²⁾ It is not known what contributes to this trend: the increase in reported accidents with increasing running speed and the increase in accident occurrence at higher speeds, the more severe outcomes of accidents that occur at higher speeds, or a mixture of both causes. Section 3.3 provides discussion of the frequency-severity indeterminacy. Speed and accident severity are discussed in more detail in Section E.2.

767
768

Exhibit E-2: Persons Injured and Property Damage per Accident Involvement by Travel Speed ⁽²²⁾

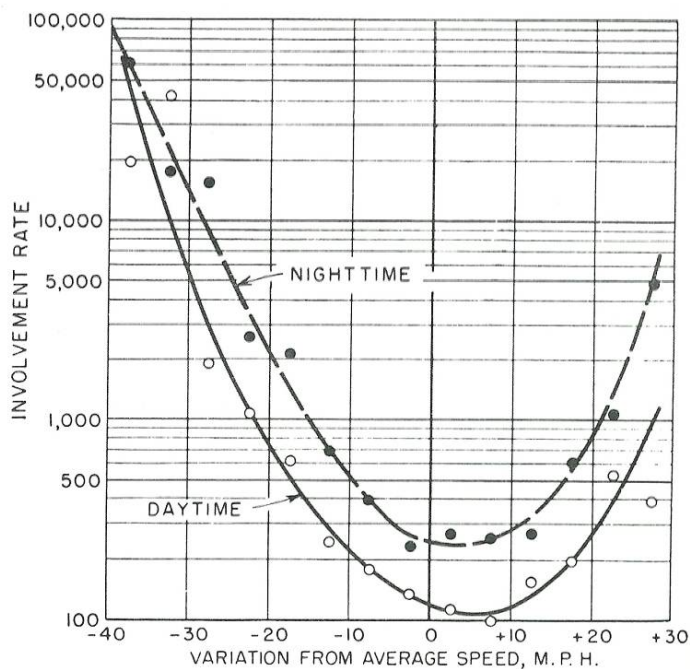


769
770

(Reproduced from Solomon's Figure 3)⁽²²⁾

771 The data can be also presented by showing the deviation from mean operating
772 speed on the horizontal axis (Exhibit E-3) instead of running speed (Exhibit E-1). The
773 curve shown in Exhibit E-3 suggests that "the greater the variation in speed of any
774 vehicle from the average speed of all traffic, the greater its chance of being involved
775 in an accident."⁽²²⁾ However, attempts by other researchers to replicate the
776 relationship between variation from mean operating speed and probability of
777 involvement by other researchers have not been successful.^(5,24,25)

778 **Exhibit E-3: Accident Involvement Rate by Variation from Average Speed**⁽²²⁾



779
780

(From Solomon's Figure 7)⁽²²⁾

781 Another consideration in the discussion of speed and probability of involvement
782 is the possibility that some drivers habitually choose to travel at less or more than the
783 average speed. The reasons for speed choice may be related to other driver
784 characteristics and may include the reasons that make some drivers cautious and
785 others aggressive. These factors, as well as the resulting running speed, may affect
786 the probability of accident involvement.

787 Although observed data do not clearly support the theory that the probability of
788 involvement in an accident increases with increasing speed, it is still reasonable to
789 believe that higher speeds and longer stopping distances increase the probability of
790 accident involvement and severity (Section E.2).

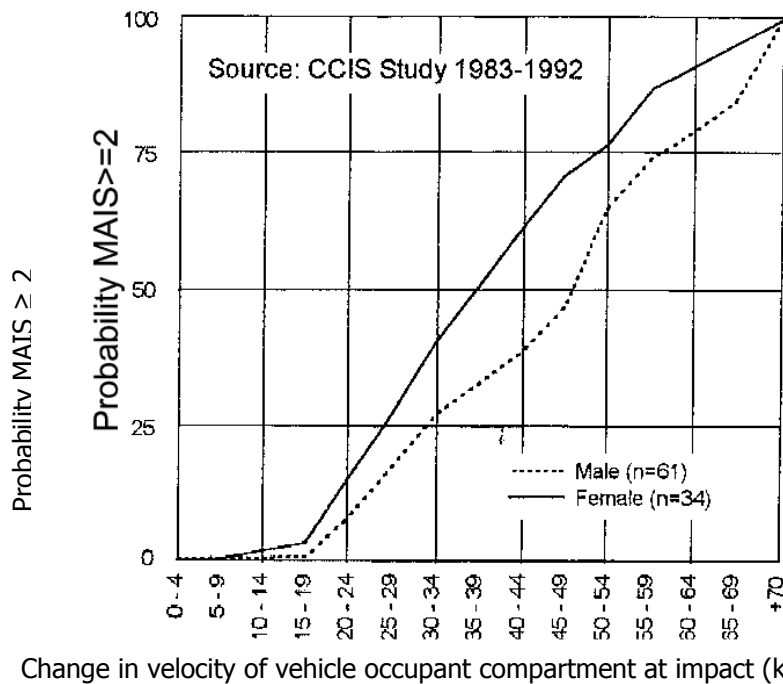
791 **E.2 Event Phase: Accident Severity and Speed** 792 **Change at Impact**

793 The relationship between the change in speed at impact and accident severity is
794 clearer than the relationship between running speed and the probability of accident
795 involvement. A greater change of speed at impact leads to a more severe outcome.
796 Damage to vehicles and to occupants depends on pressure, deceleration, change in
797 velocity and the amount of kinetic energy dissipated by deformation. All these
798 elements are increasing functions of velocity. Although vehicle speed and speed
799 distribution are commonly used, in the context of accident severity it is more
800 appropriate to use the vector 'velocity' instead of the scalar 'speed.'

801 The relationship between accident severity and change of velocity at impact is
802 strongly supported by observed data. For example, Exhibit E-4 shows the results of a
803 ten-year study of the impact of crashes on restrained front-seat occupants. Injury
804 severity is shown on the vertical axis represented by MAIS, the Maximum
805 'Abbreviated Injury Scale' (MAIS) score (An alternative way to define injury is the
806 Abbreviated Injury Scale (AIS), an integer scale developed by the Association for the
807 Advancement of Automotive Medicine to rate the severity of individual injuries. The
808 AIS scale is commonly used in detailed accident investigations. Injuries are ranked on
809 a scale of 1 to 6, with 1 being minor, 5 being severe and 6 being an unsurvivable
810 injury. The scale represents the 'threat to life' associated with an injury and is not
811 meant to represent a comprehensive measure of severity.⁽¹⁰⁾ The horizontal axis of is
812 Exhibit E-4 "the change in velocity of a vehicle's occupant compartment during the
813 collision phase of a motor vehicle crash."⁽²⁾

814 Exhibit E-Exhibit E-4 shows that the proportion of occupants sustaining a
815 moderate injury (AIS score of 2 or higher) rises with increasing change in velocity at
816 impact. The speed of the vehicle prior to the crash is unknown. For example, in a
817 crash where the change in velocity at impact is 19 mph–21 mph, about 40% of
818 restrained female front-seat occupants will sustain an injury for which $MAIS \geq 2$.
819 When the change in velocity at impact is 30-33 mph, about 75% of restrained female
820 front-seat occupants sustain such injury.⁽¹⁶⁾

821 **Exhibit E-4: Probability of Injury to Restrained Front-Seat Occupants by Change in**
 822 **Velocity of a Vehicle’s Occupant Compartment at Impact (Adapted from**
 823 **Mackay)⁽¹⁶⁾**

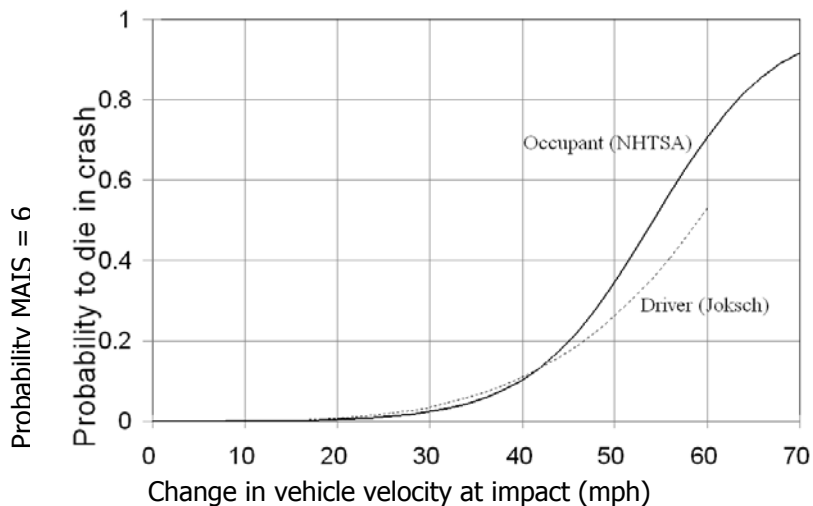


824
 825 Exhibit E-5 illustrates another example of the relationship between the change in
 826 velocity at impact and accident severity. This Exhibit illustrates data collected for two
 827 studies. The dashed line labeled Driver (Joksch) is based on a seven year study of the
 828 proportion of passenger car drivers killed when involved in accidents.⁽⁷⁾ The solid
 829 line labeled Occupant (NHTSA) is based on equations developed to calculate the risk
 830 probability of injury severity based on the change in velocity for all MAIS = 6 (the
 831 fatal-injury level).⁽²⁰⁾

832 Observed data show that accident severity increases with increasing change in
 833 velocity at impact.

834
835

Exhibit E-5: Probability of Fatal Injury (MAIS = 6) to Drivers or Occupants by Change in Vehicle Velocity at Impact^(7,20)



836

837 **E.3 Crash Frequency and Average Operating**
838 **Speed**

839 The overall relationship between safety and speed is difficult to state based on
840 observed data, as discussed in the previous sections. The effect of changes in the
841 average speed or the variance of the speed distribution on accident probability is well
842 established. This section discusses the relationship between crash frequency and
843 changes in the average operating speed of a road.

844 For fatal accidents, the change in safety is the ratio of the change in average
845 operating speed to the power of 4 (Equation D-1). This result is based on several
846 studies of roadways where the average operating speed changed from “before” to
847 “after” time periods.^(18,19)

848
$$\frac{N_1}{N_0} = \left(\frac{\bar{V}_1}{\bar{V}_0} \right)^\alpha \tag{D-1}$$

849 Where,

850 N_0 = crash frequency of the roadway before;

851 N_1 = crash frequency of the roadway after;

852 \bar{V}_0 = average operating speed of a roadway before;

853 \bar{V}_1 = average operating speed of a roadway after ;

854 α = 4 for fatal accidents;

855 α = 3 for fatal & serious injury accidents ;

856 α = 2 for all injury accidents.

857

858 Additional estimated values for the exponent α are shown in Exhibit E-6.

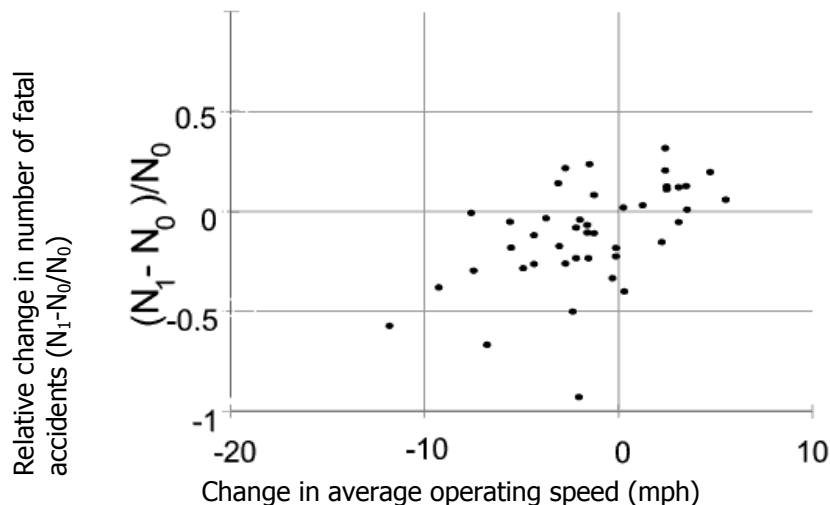
859 **Exhibit E-6: Estimates of α (exponent in Equation D-1)**

Severity	Estimate of α	95% Confidence Interval
Fatalities	4.5	4.1-4.9
Seriously Injured Road Users	2.4	1.6-3.2
Slightly Injured Road Users	1.5	1.0-2.0
All Injured Road Users (Including Fatally)	1.9	1.0-2.8
Fatal Accidents	3.6	2.4-4.8
Serious Injury Accidents	2.0	0.7-3.3
Slight Injury Accidents	1.1	0.0-2.4
All Injury Accidents (Including Fatal)	1.5	0.8-2.2
PDO Accidents	1.0	0.0-2.0

860

861 Exhibit E-7 illustrates fatal accident data from a study of 97 published studies
 862 containing 460 results for changes in average operating speed.⁽³⁾ For most roads
 863 where the average operating speed increased, the number of fatal accidents also
 864 increased, and vice versa. As can be seen in Exhibit E-7, there is considerable noise
 865 (variation) in the data. This noise (data variation) reflects three issues: the
 866 randomness of accident counts, the variety of circumstances under which the data
 867 were obtained, and the variety of causes of changes in average operating speed.

868 **Exhibit E-7: Change in Average Operating Speed vs. Relative Change in Fatal Accidents⁽³⁾**



869

870 Exhibit E-8 summarizes Accident Modification Factors (AMFs) for injury and
 871 fatal accidents due to changes in average operating speed of a roadway.⁽¹¹⁾ For
 872 example, if a road has an average operating speed of 60 mph ($\bar{V}_0 = 60$ mph), and a
 873 treatment that is expected to increase the average operating speed by 2 mph ($\bar{V}_1 - \bar{V}_0$
 874 = 2 mph) is implemented, then injury accidents are expected to increase by a factor of
 875 1.10 and fatal accidents by a factor of 1.18. Thus, a small change in average operating
 876 speed can have a large impact on crash frequency and severity.

877 The question of whether these results would apply irrespective of the cause of
 878 the change in average speed cannot be well answered at this time. If the change in
 879 crash frequency reflects mainly the associated change in severity, then the AMFs in
 880 Exhibit E-8 apply

881 **Exhibit E-8: Accident Modification Factors for Changes in Average Operating Speed⁽¹¹⁾**

Injury Accidents	\bar{V}_0 [mph]						Fatal Accidents	\bar{V}_0 [mph]					
	$\bar{V}_1 - \bar{V}_0$ [mph]	30	40	50	60	70		80	$\bar{V}_1 - \bar{V}_0$ [mph]	30	40	50	60
-5	0.57	0.66	0.71	0.75	0.78	0.81	-5	0.22	0.36	0.48	0.58	0.67	0.75
-4	0.64	0.72	0.77	0.80	0.83	0.85	-4	0.36	0.48	0.58	0.66	0.73	0.80
-3	0.73	0.79	0.83	0.85	0.87	0.88	-3	0.51	0.61	0.68	0.74	0.80	0.85
-2	0.81	0.86	0.88	0.90	0.91	0.92	-2	0.66	0.73	0.79	0.83	0.86	0.90
-1	0.90	0.93	0.94	0.95	0.96	0.96	-1	0.83	0.86	0.89	0.91	0.93	0.95
0	1.00	1.00	1.00	1.00	1.00	1.00	0	1.00	1.00	1.00	1.00	1.00	1.00
1	1.10	1.07	1.06	1.05	1.04	1.04	1	1.18	1.14	1.11	1.09	1.07	1.05
2	1.20	1.15	1.12	1.10	1.09	1.08	2	1.38	1.28	1.22	1.18	1.14	1.10
3	1.31	1.22	1.18	1.15	1.13	1.12	3	1.59	1.43	1.34	1.27	1.21	1.16
4	1.43	1.30	1.24	1.20	1.18	1.16	4	1.81	1.59	1.46	1.36	1.28	1.21
5	1.54	1.38	1.30	1.26	1.22	1.20	5	2.04	1.75	1.58	1.46	1.36	1.27

882 NOTE: Although data used to develop these AMFs are international, the results apply to North American
 883 conditions.

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